# Heat Transfer

Unit II

## Transient Heat Conduction



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- When heat transfer takes place from a body/ material, its temp changes. When temp of the body is function (fn) of location & time i,e. T(x,t), heat transfer process is called under Unsteady state conditions.
- When heat energy flows in or out of a body, its internal energy increases or decreases, which is indicated by increase or decrease in its temp. When temp of the body is a fn of time, heat transfer process is known to be taking place under Transient conditions.



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- Rate of heat transfer depends on temp gradient, and since temp changes with time, heat flow rate also changes with time continuously.
- Under transient conditions, characteristic equation for heat flow can be written as:

Rate of heat flow out Q=Rate of change of internal energy of the substance
$= - mC_p \, dT/dt$

- When heat flows out from a body, its surface temp changes. Thus a temp gradient is established from centre of the body to the surface.



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- Generally, two types of problems are encountered:

1. When body has negligible internal temp gradient (ITG) <5%
2. When body has considerable ITG (>5%)

- To decide whether ITG is <5% (can be neglected)? Here, Biot No is defined. Bi No is measure of ITG
- If Biot No (Bi) is < 0.1; then ITG will be <5%

$$Bi = \frac{hL}{k} = \frac{hL.A}{k.A} = \frac{\frac{L}{kA}}{\frac{1}{hA}} = \frac{Conductive\ Resistance\ of\ the\ object}{Convective\ Resistance\ at\ the\ surface\ of\ Object}$$







- Another Dimensionless Number utilized in transient heat transfer conditions is Fourier's No ( Fo )
- Fo is dimensionless time, which is a measure of heat conduction compared to heat storage of a body

$$Fo = \frac{k}{\rho C_p}\frac{t}{L^2} = \frac{Heat\ Conduction}{Heat\ Storage\ with\ Time}$$

Where L is characteristic length of the object/body and given as:

L=V/A; where V is the volume of the body and A is the surface area of the body







Characteristic Lengths:

Sphere: $L = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3};\quad R = Radius\ of\ Sphere$

Cylinder $L = \frac{\pi R^2 L}{2\pi RL} = \frac{R}{2}; R = Radius\ of\ Cylinder$

Cube $L = \frac{l^3}{6l^2} = \frac{l}{6};\quad l = Length\ of\ Cube$

Plate $L = \frac{A\Delta x}{2A} = \frac{\Delta x}{2};\quad \Delta x = Thickness\ of\ Plate$





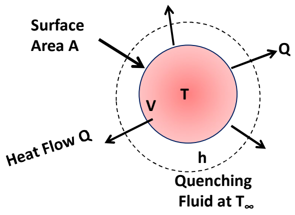


Practical Examples of Unsteady State Heat Transfer

1. Heat treatment of metals
2. Starting and shutting down of any Heat Transfer equipment like Lab Equipment
3. Starting & shutting down of engines/motors
4. Starting & shutting down of Electric Furnace
5. Starting & shutting down of Electric heater

## Quenching of Billet by Lumped Heat Capacity Method (For Heat Treatment)

- Consider a solid of volume V and surface area A, initially at temp $T_i$, suddenly placed in a fluid at temp $T_\infty$ ($T_i > T_\infty$)
- Lumped heat capacity of the solid will be $mC_p = \rho V C_p$. (Lump of heat energy is the heat required to raise/lower temp of mass m by 1°)



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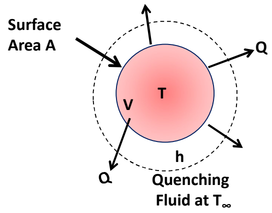
- Heat flow from billet surface of area A at any time t can be given as:

$$Q = -mC_p \frac{dT}{dt} = -\rho V C_p \frac{dT}{dt} = hA(T - T_\infty)$$




- Putting $\Theta = T - T_\infty$, the excess temp of solid above fluid, equation becomes:

$$-\rho V C_p . \frac{d\theta}{dt} = hA\theta$$



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$$-\rho V C_p . \frac{d\theta}{dt} = hA\theta; \quad OR \quad \frac{d\theta}{\theta} = \frac{-hA}{\rho C_p V} . dt$$

$$Integrating, We\ have: \quad \ln\theta = -\frac{hA}{\rho C_p V} . t + C;$$

where C is Const of integration

$$Initial \quad Conditions \quad : At\ t = 0;\ T = T_i;$$

$$hence \quad \theta = \theta_i = (T_i - T_\infty)$$

$$\Rightarrow C = \ln \theta_i$$



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$$Hence \quad \ln\theta = -\frac{hA}{\rho C_p V} . t + \ln\theta_i$$

$$\Rightarrow \ln\left(\frac{\theta}{\theta_i}\right) = -\frac{hA}{\rho C_p V} . t$$

$$\Rightarrow \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho C_p V} . t}$$



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$$Now\ \frac{hA}{\rho C_p V}.t = \frac{h}{\rho C_p L}.t \Rightarrow \left(\frac{hL}{k}\right)\left(\frac{k}{\rho C_p L^2}.t\right)$$

$$= \left(\frac{hL}{k}\right)\left(\frac{\alpha}{L^2}.t\right) = Bi.Fo\ ;\ Hence\ \Rightarrow \frac{\theta}{\theta_i} = e^{-Bi.Fo}$$

$$For\ Plate\ of\ thickness\ \Delta x;\ L = \frac{\Delta x}{2}$$

$$\left(\frac{hL}{k}\right)\left(\frac{\alpha.t}{L^2}\right) = \left(\frac{h.\frac{\Delta x}{2}}{k}\right)\left(\frac{\alpha.t}{\left(\frac{\Delta x}{2}\right)^2}\right) = Bi.Fo$$







## Instantaneous Rate of Heat Transfer

Instantaneous heat flow rate:

$$Q = h.A(T - T_\infty)\ \ and\ \ \frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho C_p V}.t}$$

$$Hence\ \ Q = h.A\left[(T_i - T_\infty).e^{-\left(\frac{h.A}{\rho.C_p.V}\right).t}\right]$$







1. $For\ Plate \Rightarrow \frac{\theta}{\theta_i} = e^{-Bi.Fo}$
2. $For\ Cylinder \Rightarrow \frac{\theta}{\theta_i} = e^{-2BiFo}$
3. $For\ Sphere \Rightarrow \frac{\theta}{\theta_i} = e^{-3BiFo}$
4. $For\ Cube\ of\ side\ L \Rightarrow \frac{\theta}{\theta_i} = e^{-6BiFo}$







## Time Constant & Response of Thermocouple

- For measurement of temp of a fluid by thermocouple, thermocouple junction should attain fluid temp as quickly as possible.(Case of unsteady state heat transfer to thermocouple junction). If time taken to attain fluid temp is small, response of thermocouple is fast.

$$\frac{\theta}{\theta_i} = \frac{T_\infty - T}{T_\infty - T_i} = e^{-\frac{hA}{\rho C_p V}.t}\ :Here\ requirement\ is\ that$$

$$\theta = (T_\infty - T)\ should\ approach\ zero\ as\ quickly$$

$$as\ possible\ for\ fast\ response\ of\ Thermocouple.$$

$$If\ we\ define\ a\ term\ Time\ Constant\ as\ \tau = \frac{\rho C_p V}{hA};$$

$$Then \Rightarrow \frac{\theta}{\theta_i} = e^{-\frac{t}{\tau}} = \frac{1}{e^{\frac{t}{\tau}}}$$

$$For\ \theta \to 0;\quad \tau\ should\ be\ as\ small\ as\ possible$$

$$For\ convenience\ if\ we\ put\ \frac{t}{\tau} = \frac{hA.t}{\rho C_p V} = 1$$

$$Then \Rightarrow \frac{\theta}{\theta_i} = e^{-1} = 0.368;$$

$$Hence\quad \theta = 0.368\theta_i$$







$$Hence\quad \theta = 0.368\theta_i$$

- Therefore, time required by the thermocouple to achieve 63.2% of initial temp difference, is called Time Constant of Thermocouple

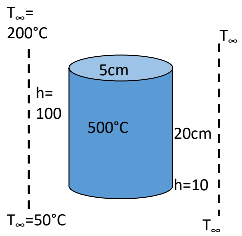
- Time Constant should be as small as possible for better response of thermocouple







Q4: A solid cylinder of steel of 5cm dia and 20cm length, initially at a uniform temp of 500°C is suddenly placed in a fluid at 200°C with h=100 W/m²K. After a period of 5 minutes, cylinder is taken out from this fluid and immediately immersed in another fluid at 50°C with h=10W/m²K. Steel properties are: $C_p$=0.46kJ/kgK; ρ=7800kg/m³; K=35W/mK. Calculate the temp of cylinder when it was taken out from the first fluid and total time required for it to achieve the temp of 100°C.




After 5 min, temp of cylinder?

Total time required for cylinder to achieve 100°C ?

Solution: Check Biot No Bi=hL/k <0.1